Experimental Modelling of Wheel rut Depth Effect of on Vehicle Velocity in wet Natural Terrain

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Abstract. *This paper presents the results of an investigation into the effect of wheel rut depth on the vehicle velocity output at a given vehicle speed selection in wet clay and sand terrain beds. This experimental investigation was carried out using a highly modified and instrumented wheeled mobility scooter renamed MOBILITY SF-3713. This vehicle was run on hard ground in order to obtain benchmark results. It was then run on sand and clay terrain test beds under controlled laboratory conditions. Results from this experiment indicate that flexible tyres can behave like rigid wheels when highly inflated and operating in soft wet deformable terrain. Further analysis of results indicate that terrain type, applied load and tyre inflation pressure all have significant effects on rut depth which eventually has direct effect on vehicle velocity. The results suggest that accurate prediction of this relationship can assist earth moving and deformable haulage road engineers in making economic and operational decisions that affect time and cost management of wheeled plant. The results from the experiment also provided reliable verification of the mathematical model developed earlier.*

Keywords*: Wheel-soil interaction, rut depth, velocity, natural terrain*

1. Introduction

Wheel rutting is the deformation of the terrain caused by rolling wheels of a vehicle traversing in soft, wet and deformable terrain (Hambleton and Drescher, 2009). It falls under a subject called terrain mechanics, which Reid (2000) defines as the study of performance of off-road vehicles in relation to their operating environments or terrain. It is concerned with the measurement of the mechanical properties and characteristics of terrain which affect vehicle mobility and the mechanics of vehicle-terrain interaction. Increased wheel rutting implies an increased waste of energy through increased rolling resistance and through unnecessary remoulding of the soil, Gunnar Bygden et al (2003). Formation of permanent ruts by rolling wheels of off-Road Vehicles (ORVs) presents a particularly complex and challenging problem when analysed within the framework of mechanics (Hambleton and Drescher, 2009). (Muleya, 2014 and Brown & Sessions 2012) clearly state that there is no rutting experienced on paved roads and hard gravel, thereby reducing the motion resistance arising from wheel sinkage. Wheel rutting is also known to significantly affect the pulling power of the vehicle in terms of traction or drawbar-pull, (Muleya, 2014). The practical implication of this result is that high tyre inflation pressure provides the best results in terms of vehicle velocity when traversing on a paved non-deformable road.

2. Background and literature review

Since the commencement of terrain mechanics pioneered by Dr M Bekker in the 1960s, many field experiments and controlled laboratory experiments have been performed in an effort to understand the relationship between moving wheels and the soil in off-road conditions, Radforth (1993). Laboratory experimental studies have been used in terrain mechanics to verify and validate predictive mathematical and simulation model results for wheel-soil interaction related research. These experiments are very useful especially in cases where field experiments are expensive and highly restricted. The performance of a wheeled vehicle moving in wet and deformable terrain in measured using velocity, wheel rut depths and pulling power measured in form of drawbar-pull. Performance output produced by moving wheels varies depending on the existing ground conditions, wheel properties, slip/skid conditions and wheel velocity

Terrain mechanics has been used by different sectors for different investigations and applications. Wheel rut depth formation is one of the primary concerns in each off-road operation due to its effects such as increased motion resistance and reduced traction effort. Grujicic et al (2009) discusses the use of soil-interaction in the military sector through the use of the High Mobility Multi-purpose Wheeled Vehicle (HMMWV) multipurpose military vehicle.

Nguyen et al (2008) illustrates that field tests on tractors have been used to validate wheel-soil interaction research results from modelling packages in agriculture industry. Hellstrom et al (2008) equally provides an insight of what goes on in the forestry sector where field tests on transportation tractors using bogie wheels have been used to validate forest plantation research. Logging and environmental damage is a primary concern in forestry management. (Iagnemma, 2009, Gibbesch, A & Schafer, B, 2004) like many other authors illustrates the application of wheel-soil interaction in the planetary exploration missions including the curiosity rover project on planet mars.

There is adequate evidence as seen from many publications that this subject area had been widely researched and applied in the military, agricultural, forestry and exploration sectors. Despite being a heavy user of wheeled and tracked construction plant, the construction industry has not fully utilised terrain mechanics as seen from the limited number of research publications. Velocity has been selected as factor under consideration because the pricing of haulage contracts is time based.

3. Significance of the study

Many sectors such military, agricultural, construction, mining, forestry, recreation and planetary exploration utilise terrain mechanics principles in their off-road vehicle mobility, (Chiroux et al, 2005, Senatore & Sandu, 2011 and Xia, 2010). A significant amount of this analysis involves theoretical analysis and semi empirical computational analysis. Outputs that are directly affected by the deformable terrain include velocity, traction in the form of drawbar-pull and rut depth. The velocity output is one of the significant factors that directly determine the overall contract period for the construction related contracts that involve wheeled plant traversing through long deformable haulage roads. This makes velocity to be as critical as it has a huge bearing on the overall contract costs. It is for this reason that the purpose of this study is to investigate the extent to which wheel rut depth can affect vehicle velocity using physical laboratory modelling. In addition, this experimental approach provided an opportunity to obtain results from live and real time runs on the ground. The results from the experiments were also used as a successful model verification tool for computational analysis and mathematical based modelling.

4. Aim and objectives of the study

The main aim of this study is to model and quantify the effect of wheel rut depth on the velocity output of the wheeled vehicles or plant operating in wet and deformable terrain using the laboratory experimental approach.

The following were the objectives outlined in order to successfully achieve the main aim above:

a) To develop the MOBILITY SF-3713, a special modified and instrumented vehicle that runs on flexible/ pneumatic tyres.

b) To model and quantify the wheel rut depth based on different tyre inflation pressures and different applied loads.

c) To model and quantify the velocity of the vehicle based on different tyre inflation pressures, different terrains and different applied loads. The three terrains include non-deformable hard ground as reference terrain, sandy terrain and clay terrain.

d) To relate the outcome of the results to the application of vehicle mobility and its associated contract period/cost in long wet deformable haulage roads

5. Research/Experiment design

The research design was based on quantitative approach in order to obtain values that would be used for comparative and isolated analysis. In order to interpret the results from the clay and sand terrain bed, the special vehicle was also run on hard non-deformable terrain in order to establish reference results/benchmarks for all other results. Running the vehicle on a pavement road produced 0mm wheel rutting thereby maintaining minimal differences between velocity outputs. The decision to adopt laboratory experiments was driven by the existing restrictive health and safety regulations that govern plant manufacturers testing sites and earth moving contracting sites. Measurements were obtained by means of instrumentation and recording physical measurements. It is worth noting that scaling and correction factors must apply to the simulated results for full scale running of wheeled plant. In addition, laboratory experiments have the advantage of controlling the consistency of the multiple runs that have to be done by changing the values of variables in order to obtain reliable isolated analysis results. The laboratory conditions provide more control of the experimental procedures such as protection from external factors which include weather, temperature and soil test bed re-building. Sand and clay terrains were selected for this experiment because they constitute the most common soil profiles. In addition these two types of soil have engineering properties that can be easily defined and measured. A total of 303 experimental runs were conducted in order to achieve outlined objectives.

6. The laboratory experiments based on MOBILITY SF-3713

The new sovereign mobility scooter was disassembled, redesigned and rebuild to meet the objectives of the study. A specially designed frame and platform was designed and mounted (Figure 1) to carry the varied weights that would represent the applied loads which were critical during the isolated analysis procedures.

6.1. MOBILITY SF-3713

The following steps below describe the sections of the modified and instrumented MOBILITY SF-3713 special vehicle used in the experiments:

A. Speed control and manoeuvring unit
B. Forward gear lever and reverse gear lever
C. Softwood timber load supporting base
D. Light metal square tubular rails for applied load side support
E. Front tyre steering spindle
F. Battery powered motor
G. Batteries for power supply.
H. Battery support base
I. Power supply cutting mechanism.
J. Rear tyre 260 x 85mm
K. Front tyre 260 x 85mm
L. Applied load of 20Kg each.
M. Camcorder stand and holder
N. Panasonic High Definition Camcorder
O. Unit for securing the camcorder

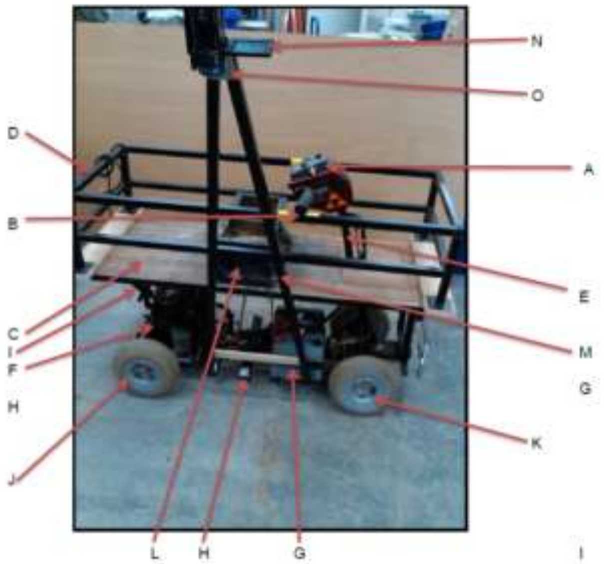

Figure 1: Special instrumented Mobility SF-3713 vehicle

6.2. Speed selection and control on the hard ground

Velocity plays a key role in determining the travel time, slip ratio and tyre performance of a wheeled vehicle or plant. In order to attain various velocities MOBILITY SF-3713 was selected because it has ten different speed selections. One speed can be selected at any given time using the speed selector as shown in figure 2. Speeds 1, 3, 5 and 7 were selected for this experiment. Speed 1 was the slowest while speed 7 was the maximum speed selected after considering the maximum length of the laboratory space used, the safety of the machine, operatives and the building.

Figure 2: Speed selection unit

6.3. Determining the actual velocity for each speed selection

Marks were made on the ground at 0 metres, 3 metres and 9 metres as distance control points. The first 3 metres was used to allow the machine to accelerate to the constant velocity. Trial runs were conducted with longer distances and 3 metres was found to be long enough to bring the machine to constant velocity. The timer was activated at 3 metres and stopped at 9 metres, travelling a total distance of 6 metres. The camcorder timer was also stopped at 9 metres. The velocity was then calculated by dividing the distance travelled in metres by the time taken in seconds. This procedure was used for all the three terrains under investigation. The advantage with this special vehicle is that it moved freely without guided rails as seen in many other studies. This gave more credibility in terms of being as close as possible to the live run environment.

6.4. Battery power meter

The vehicle had a battery meter that was used to monitor the level of the power supply in order to maintain consistency in terms of power supply to the motor as shown in figure 3 below. The power source was from two 12 volts batteries whose capacity is equivalent to 20 miles of distance coverage.

Figure 3: Battery power metre unit

6.5. Braking system of the machine

In order to bring the special vehicle to a safe stop after travelling the required distance, an electronic modification was done by introducing a cable switch. This switch was operated by means of pulling the cable from a socket in order to cut power supply. This approach created a safer stoppage mechanism of the machine without distorting the surface of the bed in the case of sand and clay terrain. Figure 4 illustrates is the cable switch and mechanism used.

Figure 4: Cable switch connected to the motor for cutting power supply

6.6. Rut depth measurement

Rut depth measurements for sand and clay terrain beds were measured in millimetres using a ruler. The rut depths were taken at various points for both tyres after which the average rut depth was established. The pattern of the wheel rutting in relation to speed, soil type, weight and tyre pressure are all recorded and compared with the benchmark results from the non-deformable hard ground experiments. The slip was also recorded from the rotational difference between rear and front tyres obtained from the camcorder (figure 9) that was mounted on the machine. The rear tyres are taken as primary measurements being the ones that were powered by the motor.

6.7. Tyre inflation pressure measurement and variation

As a way of verifying the concept surrounding the effect of tyre inflation pressure variation on the performance of the wheeled machinery, the tyres on MOBILITY SF-3713 were pneumatic and of size 260 x 85mm. Three values of tyre inflation pressure were selected for analysis namely 45PSI being the maximum, 10PSI and 3PSI. The 10PSI was selected in order to record the difference between the medium and the lowest tyre inflation pressures. The 3PSI tyre pressure was selected as the lowest working tyre pressure because the effect of tyre pressure reduction was only significant with the lowest possible tyre pressure due to the small size of the tyre. The air pressure source was industrial compressed air while the digital PCL tyre pressure gauge was used to obtain the accurate tyre pressure at each given time. Figures 5 shows the PCL tyre inflation pressure gauge used in the regulation of the tyre pressure.

Figure 5: Digital tyre pressure gauge/reader

6.8. Effect of applied load

In an effort to establish and verify the effect of applied load, four concrete cube moulds of 20kg each were used in ascending combinations to determine the cumulative effect of applied load as shown in figure 6. The MOBILITY SF-3713 vehicle was run with applied loads of 0Kg, 20kg, 40kg, 60kg and 80kg in all the three terrain profiles namely non-deformable hard ground, sand and clay terrain beds. The readings from the applied load variations were also recorded with corresponding variations in tyre inflation pressure, velocities and rut depths.

Figure 6: Two sand filled moulds on the machine acting as weights each weighing 20kg

Figure 7: Laboratory experiment sand bed showing acceleration allowance distance

Figure 7 shows the sand bed with the acceleration allowance length while figure 8 shows the clay bed. Both materials were compacted just enough to support the special vehicle and simulate terrain in its natural state.

Figure 8: Laboratory experiment clay bed

Figure 9: Camcorder fitted to the stand on Mobility SF-3713 to record travel time and wheel slip differences

7.0. Soil parameters

The timber and steel formwork for the sandy terrain bed was 3 metre long, 1 metre wide and 0.20metres high. The sand ordered had moisture content of 6.3% in order to obtain the credible results that would be consistent with the outlined objectives. The sand bed mould was filled with sand in layers of 50mm which was gently compacted just enough to simulate a realistic natural terrain that would be representative of the moist dense sandy terrain. The sand and clay terrain beds were prepared every time after running the experiment in order to create a bed of undisturbed terrain. Sand was much easier to manage in preparation, handling and disposal due to its non-cohesive nature.

MOBILITY SF-3713 was also to run on the dry, loose and un-compacted sand in order to confirm the behaviour of the machine when operated in uncompacted sand. The vehicle slipped continually without moving forward. The levelling of the sand in the sand bed was achieved by marking a horizontal line representing a uniform height around the formwork. The horizontal level was achieved by tamping the top of the material with a flat piece of timber. The internal friction angle of sand was found to be 31^{o} which is consistent with average for soil of this nature.

The clay material used was equally compacted enough to represent natural terrain. It had moisture content of 31% and a bulk density of $1.98Mg/m^3$. The soil cohesion established through tri-axial machine was $74KN/m^2$.

8.0. Results and discussion from the experiments

The section below presents the results from experiments showing the relationship between wheel rut depth and velocity output of the special vehicle. The runs are characterized by various applied loads, two tyre inflation pressure values and three types of terrain as stated in section 5 of this paper.

8.1. Rut depth and velocity on hard ground

From figure 13, the rut depth for all experiments on the ground was 0mm simply because the ground is not deformable. The effect of the weight however is seen in the changes in the tyre foot print which also signifies tyre deflection and contact area. An illustration of the respective tyre foot prints in relation to tyre inflation pressure and applied load is given in figure 10 showing the image for the fully inflated tyre on a hard surface and partially inflated tyre pressure on a hard non-deformable terrain.

The corresponding velocities are given in figure 14 on hard non-deformable ground which shows that minimal velocity variation with the highest speed selection. The highest velocity attained is with combination of the highest tyre inflation pressure of 45PSI with 0N applied load resulting in a velocity of 1.61m/s. The lowest velocity is attained from the combination of 3PSI and the maximum load of 800N giving a velocity of 1.32m/s. This is because the larger contact area reduces the velocity due to resistance. The middle loads also fall in the middle of the graphical analysis as seen in figure 14. The maximum velocity at highest load still provides higher velocity of 1.53m/s. For the hard ground fully inflated tyres gives better velocity and reduced energy loss. Figure 10 demonstrates the foot print between the smallest contact area and the largest contact area which all have significant influence on the velocity output of the wheeled machine.

8.2. Rut depth and velocity on sandy terrain bed

The deepest wheel rut depth in the sand terrain was 47mm and it was the one from the combination of 45PSI tyre inflation pressure with maximum applied load of 800N as seen from the graph in figure 15. This rut depth produced the slowest velocity of 0.35m/s at speed selection 7 as seen in figure 16. The most shallow rut depth was from all low tyre pressure tyres with 3PSI in combination with all loads, with 0N load producing the fastest velocity at 1.71m/s. Additional load provided more grip (traction) and stability resulting in the energy turning the wheel rather digging into the ground. Lower tyre pressure and wider tyre/terrain contact area provides better performance through reduced rutting. Figure 11 and figure 12 shows the behaviour of the tyre with maximum tyre pressure and minimum tyre pressure respectively all subjected to the maximum load in the sand terrain bed. This result suggests that wheeled and loaded vehicles/plant provides poor power output/efficiency in the sandy soil bed if the tyres are fully inflated with small tyre-terrain contact area.

8.3. Rut depth and velocity in clay terrain bed

Figure 17 shows experiments results for wheel rut depths arising from all loads and tyre inflation pressures in the clayey terrain. The deepest rut depth in the clay terrain (30mm) comes from maximum load of 800N and tyre pressure of 45PSI. This combination produces the lowest velocity at 1m/s at speed selection 7. The shallowest rut depth arises from the combination of 0N applied load and both tyre inflation pressures of 45PSI and 3PSI. This result confirms that applied load plays a key role in the efficiency of velocity. 45PSI produces a velocity of 1.31m/s while 3PSI produces a velocity of 1.27m/s. The velocity reduces as applied load increases although this action improves traction, a subject to be discussed and presented in the next paper. Corresponding velocities are given in figure 18.

The 400N load in both lower and higher tyre inflation pressure provide the most economic velocity as seen from the graph in figure 18 because it carries an average weight as applied load. Running wheeled and unloaded vehicles or plant throughout is not acceptable because it does not achieve any economic value.

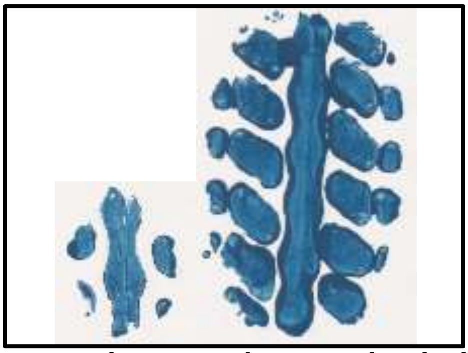

Figure 10: Tyre foot print with 45PSI and 0N load on the left pane and on the right it is the tyre foot print with 3PSI and 800N applied load. Note the difference in contact area

Figure 11: Deepest rut depth illustration in sand bed terrain with the vehicle running on maximum speed, maximum tyre inflation pressure 45PSI and maximum load 800N

Figure 12: Lowest rut depth illustration in sand terrain bed with the machine running on maximum speed, minimum tyre inflation pressure 3PSI and maximum load 800N

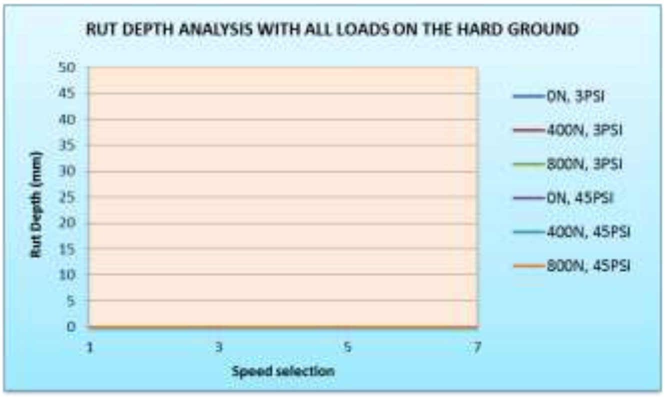

Figure 13: Wheel rut depth analysis on hard ground

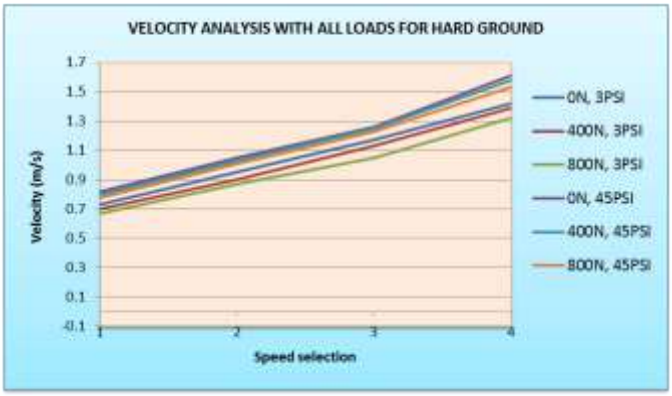

Figure 14: Corresponding velocity analysis on hard ground

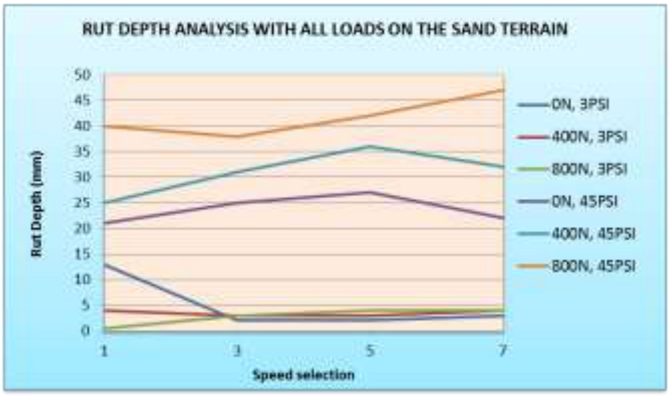

Figure 15: Wheel rut depth analysis on sandy terrain bed

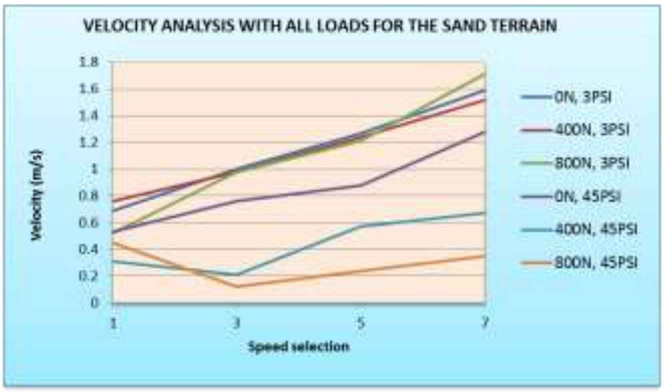

Figure 16: Corresponding velocity analysis on the sandy terrain bed

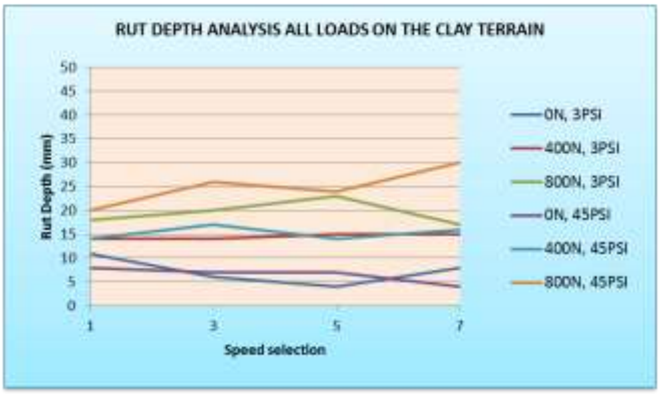

Figure 17: Wheel rut depth analysis on clay terrain bed

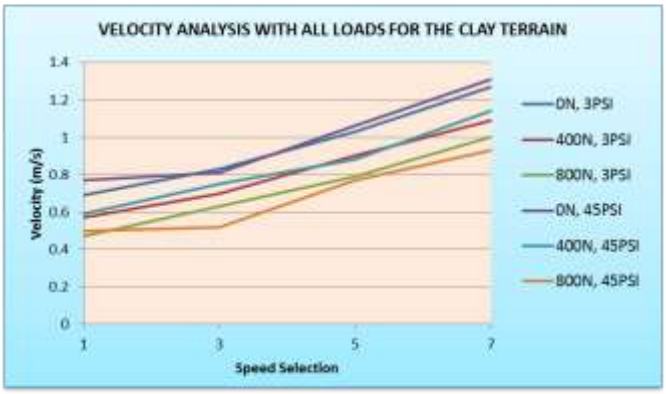

Figure 18: Corresponding velocity analysis on the clay terrain bed

All the results presented and discussed in this paper were found to be consistent with the mathematical model that was developed earlier, with up to 95% correlation in some cases.

9. Conclusions and recommendations

From the literature review and the experimental results in this study, the following conclusions and recommendations have been drawn:

- The difference of velocity generated from combinations of tyre inflation pressure and applied combination is very minimal when running on the hard ground, although highly inflated tyres provide higher velocity output due to reduced contact area resulting in reduced resistance.

- The maximum velocity is attained on the hard ground is from two combinations namely: maximum tyre pressure with minimum load. The minimum load produces the lowest contact area thereby reducing motion resistance. This run is uneconomical because it does not have economic value. Vehicles are normally expected to carry significant load during haulage trips.

- The lowest velocity is attained in the wet sandy terrain bed is from the combination of highest possible tyre pressure and maximum load operation. This combination also produces the highest rut depth creating maximum motion resistance

- The maximum velocity is attained in the wet sandy terrain bed is from two combinations namely: Lowest tyre pressure with maximum load. This combination also produces the shallowest rut depth creating minimum motion resistance. Dry sand does not support easy vehicle mobility.

- Interestingly for the wet clayey terrain bed the lowest velocity is attained from the combination of the maximum load and both high/low inflation pressure. The load is more critical in both high and low inflation pressures. This combination also produces the highest rut depth in both cases.

- Still with wet clayey terrain bed, the highest velocity is attained from the combination of the minimum load and both high/low inflation pressure. The load is more critical in both high and low inflation pressures. This combination also produces the shallowest rut depth in both cases. Economical combinations lie in the middle of the maximum and minimum velocity. Dry clay behaves opposite to dry sand as it provides adequate support for vehicle mobility.

- The results indicate that an economic/optimum rut depth and velocity exist in wheeled vehicle mobility traversing in wet and deformable terrain.

- The results further indicate that economic and optimum performance of wheeled vehicles traversing in long deformable haulage roads can be predicted. This would be very useful in time and cost management of such contracts.

- The study also suggests that effect of wheel rut depths and its effects on velocity of the vehicle cannot be ignored. Understanding of the terrain is critical in the planning and successful execution of such works.

- Although physical simulation of true ground conditions in laboratory conditions is a challenge, the results were consistent with the mathematical model results. Scaling and correction factors are required when the models are applied to full scale analysis. Selected field tests on the other hand may be required to provide provided realistic, practical data in order to validate some sections of the mathematical, simulation and laboratory based models.

- Two disadvantages associated with field test are the costs and the limited number of possible test configurations. Field tests are also highly restricted by the health and safety regulations. This laboratory experiment demonstrated that it has more advantages in terms of number of configurations and unrestricted health and safety regulations.

- Results and analysis show that while Laboratory and field tests can be used for model verification results, they can be complimented with computer simulations or computational analysis in order to overcome limitations associated with challenging terrain mechanics parameters such as wet deformable soil.

- Successful modeling of wheel-terrain parameters will lead to the elimination of overestimating and understating of cost and time variations results that from changing ground conditions for vehicles traversing in long haul wet deformable roads. This is particularly useful on haulage contracts involving heavy and medium duty wheeled equipment.

- The discussion on the effect of rut depth and velocity in relation to the ultimate pulling power (drawbar-pull) will be presented in the next paper.

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